

General Certificate of Education
January 2007
Advanced Level Examination



MATHEMATICS
Unit Further Pure 4

MFP4

Wednesday 31 January 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Show that the system of equations

$$\begin{aligned}x + 2y - z &= 0 \\3x - y + 4z &= 7 \\8x + y + 7z &= 30\end{aligned}$$

is inconsistent.

(4 marks)

2 (a) Show that $(a - b)$ is a factor of the determinant

$$\Delta = \begin{vmatrix} a & b & c \\ b + c & c + a & a + b \\ bc & ca & ab \end{vmatrix} \quad (2 \text{ marks})$$

(b) Factorise Δ completely into linear factors.

(5 marks)

3 The points P , Q and R have position vectors \mathbf{p} , \mathbf{q} and \mathbf{r} respectively relative to an origin O , where

$$\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} -3 \\ 4 \\ 20 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 9 \\ 2 \\ 4 \end{bmatrix}$$

(a) (i) Determine $\mathbf{p} \times \mathbf{q}$.

(2 marks)

(ii) Find the area of triangle OPQ .

(3 marks)

(b) Use the scalar triple product to show that \mathbf{p} , \mathbf{q} and \mathbf{r} are linearly dependent, and interpret this result geometrically.

(3 marks)

4 The matrices $\mathbf{M}_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\mathbf{M}_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ represent the transformations A and B respectively.

(a) Give a full geometrical description of each of A and B. (5 marks)

(b) Transformation C is obtained by carrying out A followed by B.

(i) Find \mathbf{M}_C , the matrix of C. (2 marks)

(ii) Hence give a full geometrical description of the single transformation C. (2 marks)

5 (a) Find, to the nearest 0.1° , the acute angle between the planes with equations

$$\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = 2 \quad \text{and} \quad \mathbf{r} \cdot (2\mathbf{i} + 12\mathbf{j} - \mathbf{k}) = 38 \quad (4 \text{ marks})$$

(b) Write down cartesian equations for these two planes. (2 marks)

(c) (i) Find, in the form $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$, cartesian equations for the line of intersection of the two planes. (5 marks)

(ii) Determine the direction cosines of this line. (2 marks)

6 (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} \quad (6 \text{ marks})$$

(b) (i) Write down a diagonal matrix \mathbf{D} , and a suitable matrix \mathbf{U} , such that

$$\mathbf{X} = \mathbf{UDU}^{-1} \quad (2 \text{ marks})$$

(ii) Write down also the matrix \mathbf{U}^{-1} . (1 mark)

(iii) Use your results from parts (b)(i) and (b)(ii) to determine the matrix \mathbf{X}^5 in the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a , b , c and d are integers. (3 marks)

Turn over for the next question

Turn over ►

- 7 The transformation S is a shear with matrix $\mathbf{M} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$. Points (x, y) are mapped under S to image points (x', y') such that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

- (a) Find the equation of the line of invariant points of S . *(2 marks)*
- (b) Show that all lines of the form $y = x + c$, where c is a constant, are invariant lines of S . *(3 marks)*
- (c) Evaluate $\det \mathbf{M}$, and state the property of shears which is indicated by this result. *(2 marks)*
- (d) Calculate, to the nearest degree, the acute angle between the line $y = -x$ and its image under S . *(3 marks)*

- 8 The matrix $\mathbf{P} = \begin{bmatrix} 4 & -1 & 2 \\ 1 & 1 & 3 \\ -2 & 0 & a \end{bmatrix}$, where a is constant.

- (a) (i) Determine $\det \mathbf{P}$ as a linear expression in a . *(2 marks)*
- (ii) Evaluate $\det \mathbf{P}$ in the case when $a = 3$. *(1 mark)*
- (iii) Find the value of a for which \mathbf{P} is singular. *(2 marks)*
- (b) The 3×3 matrix \mathbf{Q} is such that $\mathbf{PQ} = 25\mathbf{I}$.

Without finding \mathbf{Q} :

- (i) write down an expression for \mathbf{P}^{-1} in terms of \mathbf{Q} ; *(1 mark)*
- (ii) find the value of the constant k such that $(\mathbf{PQ})^{-1} = k\mathbf{I}$; *(2 marks)*
- (iii) determine the numerical value of $\det \mathbf{Q}$ in the case when $a = 3$. *(4 marks)*

END OF QUESTIONS